



SEMESTRAL

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Álgebra

Tema: Números complejos I

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1. Determine la siguiente suma:

$$\left(\frac{1-i}{1+i}\right) + \left(\frac{1-i}{1+i}\right)^2 + \left(\frac{1-i}{1+i}\right)^3 + \dots + \left(\frac{1-i}{1+i}\right)^{35}$$

A) 0

B) 1

~~C) -1~~D) i E) $-i$

Recordar

$$\frac{1-i}{1+i} = -i$$

$$i^{4k} = 1 ; \forall k \in \mathbb{Z}$$

$$i + i^2 + i^3 + i^4 + \dots + i^{4k} = 0 ; \forall k \in \mathbb{Z}$$

Resolución

$$\text{En } \left(\frac{1-i}{1+i}\right) + \left(\frac{1-i}{1+i}\right)^2 + \left(\frac{1-i}{1+i}\right)^3 + \dots + \left(\frac{1-i}{1+i}\right)^{35}$$

$$-i + i^2 + i^3 + \dots + i^{35}$$

$$\underbrace{-i + i^2 + i^3 + \dots + i^{35}}_0 + i^{36} - i^{36}$$

$$-1$$

2. Determine n , para que z sea un complejo real, si

$$z = \frac{n + 5i}{1 + 2i} + \frac{1 + i}{1 - i}$$

A) 6

B) 1

~~C) 5~~

D) 2

E) 3

Recordar

$$\frac{1 + i}{1 - i} = i$$

$$i^2 = -1$$

Sea: $z = \frac{a + bi}{m + ni}$

z es un complejo real si:

$$an = bm$$

Resolución

$$\text{En } z = \frac{n + 5i}{1 + 2i} + \frac{1 + i}{1 - i}$$

$$z = \frac{n + 5i}{1 + 2i} + i$$

$$z = \frac{n + 5i + i + \overset{-2}{2i^2}}{1 + 2i}$$

$$z = \frac{(n - 2) + 6i}{1 + 2i}$$

Para que z sea un complejo real

$$(n - 2)\cancel{2} = \cancel{6}.1$$

$$n - 2 = 3$$

$$n = 5$$

3. Determine el valor reducido de

$$M = 2 \left| \frac{3 - 4i}{1 + i} \right| + (7 + 5i) \times \overline{(7 + 5i)} - |(7 + 5i)^*|^2$$

A) 5

B) 1

C) $\sqrt{2}$

D) $2\sqrt{2}$

E) $5\sqrt{2}$

Recordar

$$\left| \frac{z}{w} \right| = \frac{|z|}{|w|}$$

$$z \cdot \bar{z} = |z|^2$$

$$|z| = |z^*|$$

Resolución

$$\text{En } M = 2 \left| \frac{3 - 4i}{1 + i} \right| + (7 + 5i) \times \overline{(7 + 5i)} - |(7 + 5i)^*|^2$$

$$M = 2 \frac{|3 - 4i|}{|1 + i|} + |7 + 5i|^2 - |7 + 5i|^2$$

$$M = \cancel{\sqrt{2}^2} \cdot \frac{5}{\cancel{\sqrt{2}}}$$

$$M = 5\sqrt{2}$$

4. Determine la suma de todos los $z \in \mathbb{C}$ tal que

$$z + 4 = |z|^2 + 2i$$

A) $1 + 2i$

B) $3 + 2i$

C) $1 + 2i$

D) $1 + i$

~~E) $1 + 4i$~~

Recordar

Si $z \in \mathbb{C} \Rightarrow z = a + bi ; a, b \in \mathbb{R}$

$$|z| = \sqrt{a^2 + b^2}$$

$$a + bi = m + ni \Leftrightarrow a = m \wedge b = n$$

Resolución

Como $z \in \mathbb{C} \Rightarrow z = a + bi$

En $z + 4 = |z|^2 + 2i$

$$a + bi + 4 = \sqrt{a^2 + b^2}^2 + 2i$$

$$(a + 4) + bi = a^2 + b^2 + 2i$$

$$\Rightarrow a + 4 = a^2 + b^2 \wedge b = 2$$

$$a + 4 = a^2 + 4$$

$$0 = a^2 - a$$

$$0 = a(a - 1)$$

$$a = 0 \vee a = 1$$

$$\Rightarrow \begin{matrix} z_1 = 0 + 2i \\ z_2 = 1 + 2i \end{matrix} +$$

$$z_1 + z_2 = 1 + 4i$$

5. Sea z y $w \in \mathbb{C}$, tal que $|z + w| = |z - w|$
determine $\text{Re}z \times \text{Re}w + \text{Im}z \times \text{Im}w$.

A) 1

~~B) 0~~

C) 3

D) 2

E) 4

Recordar

Si $z \in \mathbb{C}$  $z = a + bi$; $a, b \in \mathbb{R}$

Donde $a = \text{Re}(z)$
 $b = \text{Im}(z)$

Si $z = a + bi \wedge w = m + ni$

 $z + w = (a + m) + (b + n)i$
 $z - w = (a - m) + (b - n)i$

$$|z| = \sqrt{a^2 + b^2}$$

$$(x + y)^2 - (x - y)^2 = 4xy$$

Resolución

Como $z, w \in \mathbb{C}$,  $z = a + bi$
 $w = m + ni$

Piden $\text{Re}z \times \text{Re}w + \text{Im}z \times \text{Im}w = am + bn$

$$\text{En } |z + w| = |z - w|$$

$$\left| (a + m) + (b + n)i \right| = \left| (a - m) - (b - n)i \right|$$

$$\sqrt{(a + m)^2 + (b + n)^2}^2 = \sqrt{(a - m)^2 + (b - n)^2}^2$$

$$(a + m)^2 + (b + n)^2 = (a - m)^2 + (b - n)^2$$

$$\underbrace{(a + m)^2 - (a - m)^2}_{4am} + \underbrace{(b + n)^2 - (b - n)^2}_{4bn} = 0$$

$$4(am + bn) = 0$$

$$am + bn = 0$$

6. Determine el área de la región que se determine al unir los afijos de z , w y p tal que

$$z = \frac{3 + 4i}{4 - 3i}; \quad w = \bar{z} + z^*; \quad p = wi + i$$

A) 2

~~B) 3~~

C) 4

D) 5

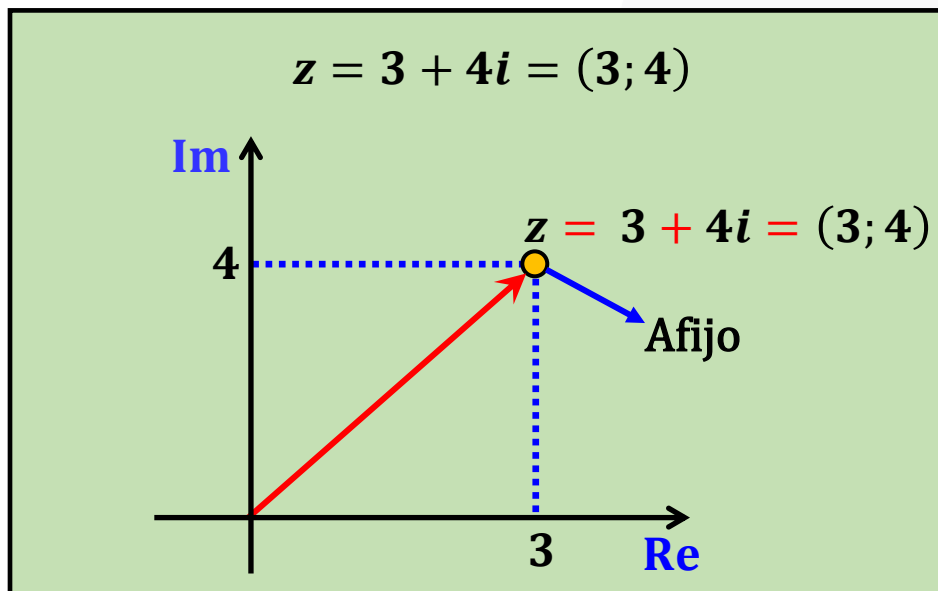
E) 6

Recordar

$$i^2 = -1$$

Sea el complejo $z = a + bi \in \mathbb{C}$, se define

$$\bar{z} = a - bi \quad z^* = -a - bi$$



Resolución

Hallamos z , w y p

$$z = \frac{3 + 4i}{4 - 3i} \rightarrow z = \frac{(3 + 4i)i}{(4 - 3i)i} = \frac{(3 + 4i)i}{4i - 3i^2} = i$$

$$w = \bar{z} + z^* = -i - i = -2i$$

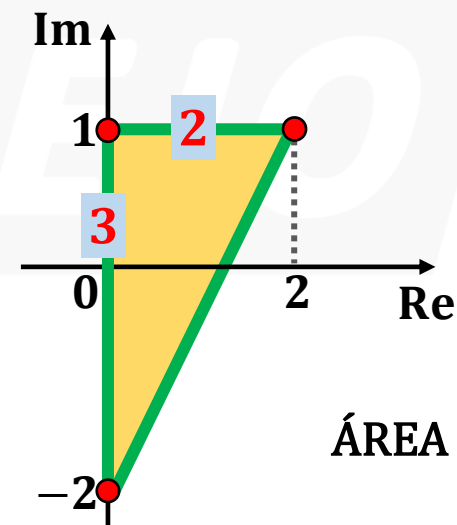
$$p = wi + i = (-2i)i + i = -2i^2 + i = 2 + i$$

Graficamos z , w y p

$$z = i = (0; 1)$$

$$w = -2i = (0; -2)$$

$$p = 2 + i = (2; 1)$$



$$\text{ÁREA} = \frac{2 \cdot 3}{2} = 3$$

7. Resuelva la ecuación

$$(|iz + 4| + |\bar{z} + 4i|)|\bar{z} + 2i| = 0; z \in \mathbb{C}$$

Dé como respuesta la suma de los módulos de las raíces.

A) 2

B) 4

~~C) 6~~

D) 8

E) 10

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$$\text{Si } ab = 0 \Rightarrow a = 0 \vee b = 0$$

$$\text{Si } |z| + |w| = 0 \Rightarrow z = 0 \wedge w = 0$$

$$\text{Si } |z| = 0 \Rightarrow z = 0$$

$$\bar{\bar{z}} = z$$

$$|z| = |\bar{z}| = |z^*|$$

Resolución

$$\text{Como } (|iz + 4| + |\bar{z} + 4i|)|\bar{z} + 2i| = 0$$

$$\Rightarrow |iz + 4| + |\bar{z} + 4i| = 0 \vee |\bar{z} + 2i| = 0$$

$$(|iz + 4| = 0 \wedge |\bar{z} + 4i| = 0) \vee \bar{z} + 2i = 0$$

$$(iz + 4 = 0 \wedge \bar{z} + 4i = 0) \vee \bar{z} = -2i$$

$$(iz = -4 \wedge \bar{z} = -4i) \vee |\bar{z}| = |-2i|$$

$$(-i \cdot iz = -i \cdot -4 \wedge \bar{\bar{z}} = \overline{-4i}) \vee |z| = |2i|$$

$$(z = 4i \wedge z = 4i) \Rightarrow |z| = 2$$

$$\Rightarrow |z| = 4$$

Piden la suma de los módulos de las raíces.

$$4 + 2 = 6$$

8. Determine la secuencia correcta de verdad (V) o falsedad (F) según corresponda:

I. Si $|z| = \bar{z} \rightarrow \text{Re}(z) \geq 0$

II. Si $zi = \bar{z} \rightarrow \text{Re}(z) + \text{Im}(z) = 0$

III. Si $z - \bar{z} = |z| \rightarrow z = 0$

~~A) VVV~~

B) VFF

C) FFF

D) FVF

E) VVF

Recordar

El $|z|$ siempre será un complejo real positivo o 0.

Es decir $|z| \geq 0 ; \forall z \in \mathbb{C}$

Si $|a| = a \rightarrow a \geq 0$

$|z| = \# \text{real} \geq 0$

$a + bi = m + ni \leftrightarrow a = m \wedge b = n$

Resolución

Como $z \in \mathbb{C} \rightarrow z = a + bi$

I. Si $|z| = \bar{z} \rightarrow \text{Re}(z) \geq 0$ (V)

$|a + bi| = a - bi \rightarrow b = 0$

$|a| = a \rightarrow a \geq 0$ o sea $\text{Re}(z) \geq 0$

II. Si $zi = \bar{z} \rightarrow \text{Re}(z) + \text{Im}(z) = 0$ (V)

$(a + bi)i = a - bi$

$ai - b = a - bi \rightarrow \begin{cases} -b = a \wedge a = -b \\ a + b = 0 \end{cases}$
 $\text{Re}(z) + \text{Im}(z) = 0$

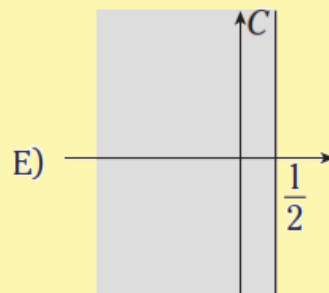
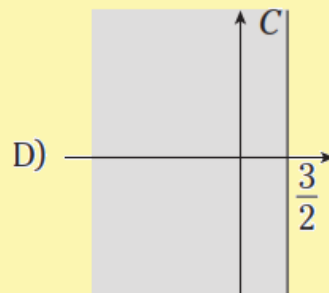
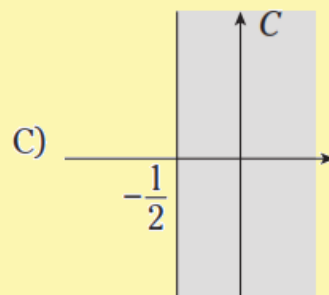
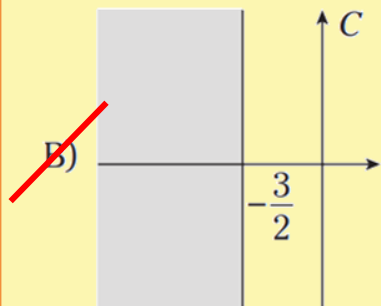
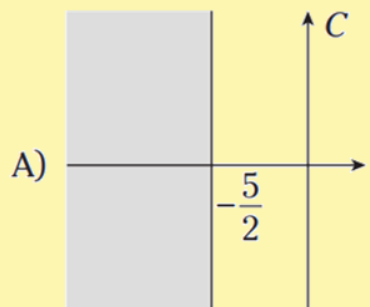
III. Si $z - \bar{z} = |z| \rightarrow z = 0$ (V)

$2bi = |a + bi| \rightarrow b = 0$
 $0 = |a| \rightarrow a = 0$
 $\left. \begin{array}{l} b = 0 \\ a = 0 \end{array} \right\} z = 0$

9. Indique la gráfica del siguiente conjunto de números complejos:

$$M = \left\{ z \in \mathbb{C} / |z + 2\bar{z}| \geq |z| \wedge \frac{|z + 2|}{|z + 1|} \leq 1 \right\}$$

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Resolución

Como $z \in \mathbb{C}$  $z = x + yi$

De las condiciones de M

$$|z + 2\bar{z}| \geq |z|$$

$$|3x - yi| \geq |x + yi|$$

$$\sqrt{(3x)^2 + (-y)^2} \geq \sqrt{x^2 + y^2}$$

$$9x^2 + y^2 \geq x^2 + y^2$$

$$8x^2 \geq 0$$

$$x^2 \geq 0$$

Siempre se cumple
(NO HAY RESTRICCIÓN)

$$\wedge \frac{|z + 2|}{|z + 1|} \leq 1$$

$$|z + 2| \leq |z + 1| \quad \wedge \quad z \neq -1$$

$$|(x + 2) + yi| \leq |(x + 1) + yi|$$

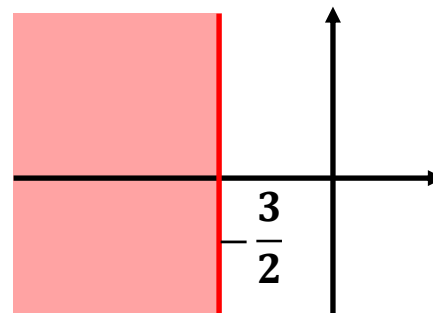
$$\sqrt{(x + 2)^2 + y^2} \leq \sqrt{(x + 1)^2 + y^2}$$

$$x^2 + 4x + 4 + y^2 \leq x^2 + 2x + 1 + y^2$$

$$2x \leq -3$$

$$x \leq -\frac{3}{2}$$

(Son todos los complejos donde su parte real es menor o igual a $-\frac{3}{2}$)



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